SYNTHESIS OF MATCHED FILTER USING PSEUDO - RANDOM SIGNALS

A Thesis submitted

in partial fulfilment of the requirements

for the Degree of

Master of Technology

by

K. R. RAM GOPAL

to the

Department of Electrical Engineering

Indian Institute of Technology

Kanpur

May 1968

This is to certify that this work on Synthesis of Matched Filter Using Pseudo Random Signals has been carried out under my supervision and it has not been submitted elsewhere for a degree.

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This thesis has been approved for the award of the Degree of Master of Technology in accordance with the regulations of the Indian Institute of Technology, Kanpur.

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ACKNOWLEDGEMENTS

The author wishes to express his deep sense of gratitude to Dr. V. P. Sinha for all guidance and encouragement provided in the course of the project. Thanks are also due to Dr. K. R. Sarma for having initiated the work.

He also acknowledges his appreciation to Dr. T. R. Viswanathan and Dr. R. N. Biswas for many valuable suggestions. Finally, thanks are due to the Electrical Engineering Department, I.I.T., Kanpur for extending laboratory and other facilities.

SYNOPSIS

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Literature on Matched Filter Theory has been reviewed and constructional details of 7 code matched filter are discussed. A digital computer program (FORTRAN) for generation of pseudo random sequences is also provided.

TABLE OF CONTENTS

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1. INTRODUCTION

1.1. Origin of Matched Filter:-

The subject of matched filter originated in the studies of maximising the signal to noise ratio at the output of a receiver.[1] Historically speaking, the idea of matched filtering, was for the first time introduced in 1943 in North's[2] study of signal to noise ratio maximisation of pulsed radar systems. However, the term "matched"[3] was first introduced in a classic paper by David Middleton and Van Vleck in 1946, during their study of a theoretical comparison of the visual aural and meter reception of pulsed signals in the presence of noise.

1.2. The Philosophy of Matched Filter:-

A very specific notion around which the whole idea of matched filtering revolves is that one wave form may be correlated with another, by,

1. passing the first waveform through a linear system whose impulse response is the time reverse of the second waveform

and 2. observing the output at a certain instant of time.

However a special case arises, which is of chief interest in this work when the two waveforms are made the same. In this case, the filter is said to be "matched" into the input waveform. The filter output as a function of real time is then the auto-correlation function of the system.

1.3. Nature of Matched Filter Output and its Utility:-

As the output of a matched filter receiver is the auto-correlation function of the input signal waveform itself, it does not preserve the shape of the received signal. But generally, in most applications of matched filter (MF), the primary point of interest is only to improve our ability to "see" or recognise a pulsed signal, in the presence of noise,[4] in which case maintaining the fidelity of the signal at the output is of no special significance. However, there are instances for example in PPM systems, applied to tracking radars, wherein pulse detection is more closely related to maintaining the shape of the input signal waveform at the output rather than just maximising the signal to noise ratio. In such cases, the techniques employed are different. One of them is the least square smoothing and prediction theory of Wiener[5]. But however, the main point of interest, in this work, is the maximisations of signal to noise ratio.

1.4. Use of Matched Filter as a Decision Device:-

One of the most important applications of MF is in radar receivers.

The ability of a radar receiver to detect a "weak" echo signal, is limited by the noise energy that occupies the same bandwidth as does the signal energy. The specification of minimum detectable signal is sometimes difficult, because of its statistical nature and also because the criterion for deciding whether a target is present or not, may not be deterministic. In such cases, matched filtering provides a basis for making a reasonable decision as to

whether a target is present or not. Consequently the overall objective of MF is, not only to maximise signal to noise ratio (SNR) but also to minimise the probability of error, in detection of signals in presence of noise.[6] It is important to note, that there is no certainty, that in every case if NSNR is maximised, the probability of error in detection will be minimised. Hence one has also to resort, to minimising the probability of error in detection in spite of maximisation of SNR. This is because detection is essentially a decision process and the decision is about whether a signal is actually present or not. It may be critical when trailing a hostile aircraft. As is now evident, there has to be some basis for making such a decision, there is what is known as "threshold", in a receiver. The decision mechanism answers for a signal only if the threshold is exceeded, otherwise not. In applications like radar, this decision must be made with minimum probability of error. An ideal solution to this problem, is a binary channel with yes - no operation based on whether or not the receiver output exceeds the threshold. The threshold should be exceeded only when signal and noise both are present and should not be exceeded when noise only is present. But however, it is quite possible that noise, which is specified by some probability distribution, can assume amplitudes exceeding the threshold and thereby operate system output circuits, when not desirable. In view of this fact, the probability of occurrence of this type of exceeding the threshold, namely "false-alarm", has to be minimised. One way of doing this is to keep a threshold, large enough to prevent false-alarm. But then weak signals may completely miss detection and hence the threshold should be such

that a compromise is effected between false-alarm rate and the so called "miss rate". The MF here again proves to be a very useful device as it can be well designed to produce any desirable output peak amplitude, facilitating a convenient choice of threshold.

1.5. Choice of Matched Signals:-

1.5.1. Their Desirability and Synthesis:

The most desirable matched signal is the one which when passed through a linear system whose impulse response is the time reverse of the signal itself, gives at the output of the filter, a sharp peak, with low clutter on either sides of the peak. "Clutter" refers to number of minor peaks surrounding the main peak. One important flexibility the MF theory provides for a designer is that he can choose any signal to be a matched signal as long as its MF is "physically realizable". Actually the problem of synthesizing an MF may be approached in two ways. One is the direct approach. In this, one is to choose a desirable signal and then to synthesize a filter matched to this signal. Another is the indirect approach wherein one has to obtain a given filter, determine its impulse response and then produce the time reverse of the impulse response which is the matched signal itself.[7] It is however the direct approach that is discussed through out this work.

1.5.2. Binary Signals:

It has been stated before, that the MF output is nothing but the autocorrelation function of the matched signal itself. From the point of view of computational ease it is found that the study of autocorrelation functions is made much hsimpler if the matched

signal amplitudes are limited only to two levels, namely + 1 and -1. Such signals are called binary signals. There are many advantages in going for this type of signals, as will be enumerated later. Considering, pulses of amplitude + 1 or - 1 having a finite time duration T, to be elementary signals, the matched signal is constructed out of a number of these, forming what is known as length of signal. If say, a matched signal has a length nT (T = time duration of elementary signal; n = number of elementary signals) and amplitude modulus as equal to unity, then the MF output will be of 2nT duration of time and will have a central peak whose amplitude is n times signal amplitude. Thus what matched filtering does, is to concentrate most of the energy, from the elementary signals forming the matched signal, into the central peak and hence facilitate detection even in presence of noise. This is exactly the meaning of maximising SNR.

1.6. Coding of Signals:-

It will be found that to obtain the desirable output namely a high peak with low clutter on either side, thee matched signal has to look as "noise like" as possible. By noise like, it only means that in the signal, the occurrence of + 1's and -1's has to appear as arbitrary as possible. Hence standard coding procedures may be adopted to discover the right type of sequences of + 1's and - 1's. The noise like requirement mentioned above, is well met by signals, adopting what are known as pseudorandom sequences. These are also known as orthogonal or pseda-noise (PN) sequences and are discussed later in greater detail. However there are other codes

for example biorthogonal, transorthogonal, Bose-Chaudhuri codes etc., which are formulated for lesser bandwidth requirement, as orthogonal codes require the maximum bandwidth. But a discussion of these, is beyond the scope of this work.

1.7. Statement of the Problem:-

The aim of the present work is to formulate a digital computer program for the generation of pseudorandom sequences and to construct a 7 code matched filter generator pair, using tapped delay lines.

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2. Theoretical Discussion

2.1. Definition of MF :-

If $s(t)$ is some physical wave form, then a filter which is matched to $s(t)$ is by definition, one with an impulse response.

$$h(\tau) = k\, s(\Delta - \tau) \qquad (1)$$

where k and Δ are arbitrary constants.

The transfer function of an MF which is Fourier transform of the impulse response, has the form

$$H(j\omega) = \int_{-\infty}^{+\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= \int_{-\infty}^{+\infty} k\, s(\Delta - \tau)\, e^{-j\omega\tau}\, d\tau$$

$$= k\, e^{-j\omega\Delta} \int_{-\infty}^{+\infty} s(\tau')\, e^{j\omega\tau'}\, d\tau' \qquad (2)$$

where $\tau' = \Delta - \tau$.

Now, Fourier transform of $s(t)$ is,

$$s(j\omega) = \int_{-\infty}^{+\infty} s(t)\, e^{-j\omega t}\, dt \qquad (3)$$

Comparing (2) and (3) it is found that,

$$H(j\omega) = k\, s(-j\omega)\, e^{-j\omega\Delta} = k\, s^{*}(j\omega)\, e^{-j\omega\Delta} \qquad (4)$$

Thus, if the amplitude and delay factor are made equal to unity my making the arbitrary constants k and Δ equal to 1 and 0 respectively, the transfer functions of the MF is the complex

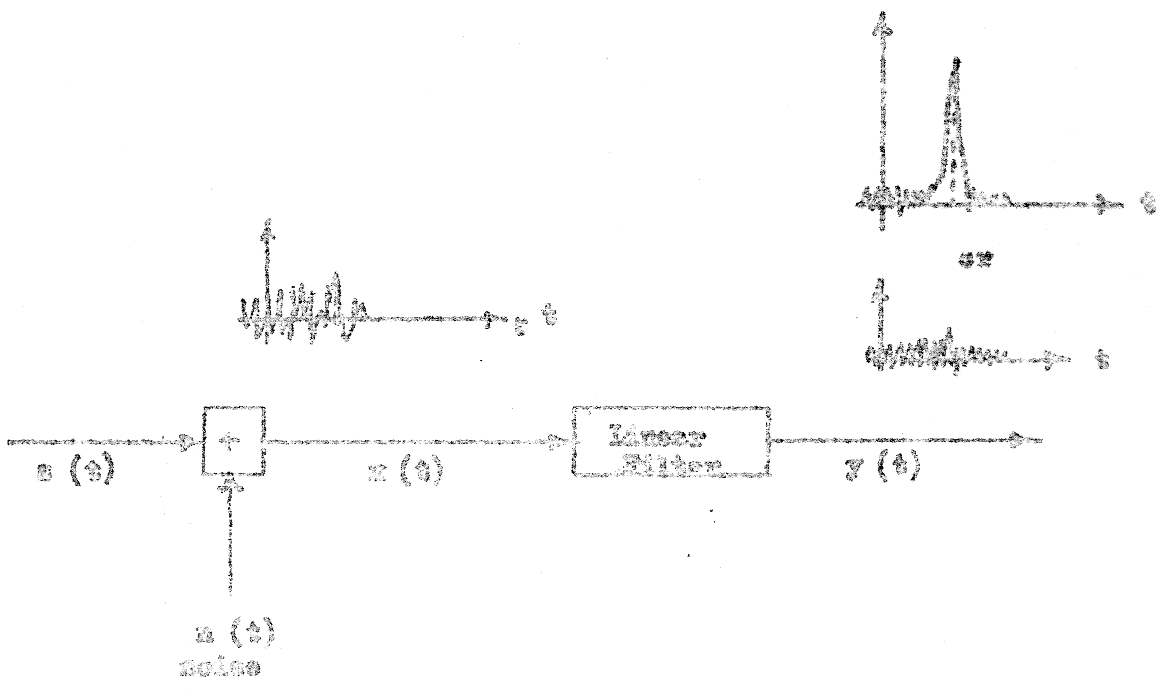


Fig. 2. (a) Maximization of signal to noise ratio.

conjugate of the Fourier transform of the signal to which it is matched. Hence MF is also called a "conjugate filter".

There are two criteria upon which MF analysis may be made, depending upon the use to which it is to be put. They are

(1) Mean square criteria

(2) Probabilistic criteria.

2.2.1. Mean Square Criteria[8]

This mainly deals with maximisation of SN power ratio. This criteria is formulated in terms of first order noise statistics only. The associated mathematics is, therefore fairly simpler and straightforward.

Whenever a waveform x (t) is received at the receiver two possibilities exist:

(a) x (t) may consist exclusively of white noise n (t)

or (b) x (t) may consist of n (t) plus a signal s (t), say a radar return of known waveform, as illustrated in figure 2.(a)

Thus the problem is to find out which one of the situations is true, in the most "unambiguous manner possible". The straight forward solution of course, is, to pass x (t) through a linear filter, in such a manner that if s(t) is present, then the filter output at some time $t = \Delta$ is considerably larger than, if s (t) is absent. Since, the filter has been assumed to be linear, its output y (t) will consist of a noise component y_n (t) due to n (t) and if s (t) is present, a signal component y_s (t) due to s (t). Hence the filter should be such, that it

makes the instantaneous power in $y_s(\Delta)$ as large as possible compared to the average power in n (t) at time Δ. Average power in n (t) is defined as the integrated power under the noise power density spectrum.

The output noise power density is $N_o/2 \, |G(j\omega)|^2$, where $G(j\omega)$ is the transfer function of the filter. Hence the output noise power is

$$\sigma^2 = \frac{N_o}{2} \int_{-\infty}^{+\infty} |G(j\omega)|^2 \, df \qquad (5)$$

The input signal spectrum is $s(j\omega)$, then $s(j\omega)\, G(j\omega)$ is the output signal spectrum. To get back $y_s(\Delta)$, one has to take an inverse Fourier transform of the output signal spectrum evaluated at time $t = \Delta$; this will be,

$$y_s(\Delta) = \int_{-\infty}^{+\infty} S(j\omega)\, G(j\omega) e^{j\omega\Delta} \, df \qquad (6)$$

The figure of merit ρ, on which the filter design is based is defined as the ratio,

$$\rho = \frac{|y_s(\Delta)|^2}{\sigma^2}$$

where σ is the mean square value of output noise. Thus, ρ, the figure of merit is nothing other than the output signal to noise ratio, that we wish to maximise and this is given by ratio of square of (6) to (5).

$$\rho = \frac{\left[\int_{-\infty}^{+\infty} S(j\omega)\, G(j\omega) \; e^{j\omega\Delta} \, df \right]^2}{\frac{N_o}{2} \int_{-\infty}^{+\infty} |G(j\omega)|^2 \, df} \qquad (7)$$

Consider the Schwarz in equality, b

$$\left| \int f(x)\, g(x)\, dx \right|^2 \leq \int |f(x)|^2\, dx \int |g(x)|^2\, dx \qquad (8)$$

by replacing G ($j\omega$) with f (x) and s ($j\omega$) $e^{j\omega\Delta}$ with g (x) in (7), it follows that

$$\rho \leq \frac{2}{N_o} \int_{-\infty}^{+\infty} |S(j\,\omega)|^2\, df \qquad (9)$$

In (8) the equality holds if and only if f (x) = k g^* (x). Also noting that s $(j\omega)^2$ is the energy of the signal, we get

$$\rho = \frac{2E}{N_o} \qquad (10)$$

By virtue of the fact f (x) = k g^* (x), it follows that

$$G\,(j\,\omega) = k\,S^*\,(j\,\omega)\; e^{-j\,\omega\,\Delta} \qquad (11)$$

Hence when filter is matched to s (t), a maximum value of ρ is obtained. Because the equality holds only when f (x) = k g^*(x), the matched filter arrived at in (11), remains the only type of linear filter, maximising ρ .

2.2.2. Probabilistic Criteria:

It should be noted that although mean square criteria discussed above is suitable for most of the purposes, it is generally preferable to use that criteria which is directly related to the performance ratings of systems such as radar and communication, in which one may be interested. The above mentioned performance ratings are usually probabilistic in nature, involving probabilities of detection of false alarm of error etc. and these are the quantities that are to be optimised. This leads to the following discussion on classical statistical hypothesis

testing and estimation theories.

The problem is however, the same as before. Namely, the observed signal x (t) is either solely due to noise, or to both an exactly known signal and noise.

Denoting noise only, by Hypothesis H_o and the other by H_1, a test has to be devised for deciding in favour of H_o or H_1.

In this case two types of errors are possible, and we are interested in the probabilities of their occurrences.

Type I -- Deciding in favour of H_1 when H_o is true

and Type II-- Deciding in favour of H_o when H_1 is true.

First one results in false-alarm and second one in missing of signal. Let the probabilities of making such errors be denoted by and respectively. For a choice of criterion if we choose to minimise the average of and , i.e. the overall probability of error, then we require to know the a priori probabilities of H_o and H_1. Or if one type of error is costlier than the other, then we may have to minimise the average cost. But when neither the a priori probabilities nor the costs are known or even definable one cannot establish the test to minimise either of these quantities. The decision rule must then be based on some different idea. An obvious basis for a decision rule is given by what is known as the "maximum likelihood principle" by which it can be inferred that a cause exists which would most probably yield the observed value. This means the relation between cause and effect is probabilistic in nature. Either a cause or effect

is definitely known but not both. For as cause known to be definite a range of possible effects can be conceived of which anyone may be correct and the problem of detection is to find this unambiguously.

Some of the statistical terms that apply to hypotheses testing are enumerated here.[9] As it is explicit by now, any decision rule for testing two hypotheses, is equivalent to partitioning an observation space X r into two parts X_o and X_1. If an observation X occurs which belongs to X_o, the rule is to choose H_o; if X belongs to X_1, the rule is to choose H_1. Thus when either of the sets X_o or X_1, is specified, the rule is completely determined because the other set is the remaining part of X, and hence the absolute need is to mention only one of them. It is also common practice to discuss hypotheses tests in terms of one single hypothesis, H_o. Thus in terms of H_o, X_o is called the " acceptance region" and X_1 is called the "rejection region" or more commonly called the "critical region". If the observation X falls in X_1, so that H_o is rejected when in fact H_o is true, it is said that an error of 'first kind' has been made; if X falls in X_o when H_1 is true, it is said that an error of 'second kind' has been made. This is illustrated in Figure 2. The probability of rejecting H_o when it is true, $P_o\ (X_1)$ is called the "level" or "size" of the test. The probability of rejecting H_o when it is false, $P_1\ (X_1)$ is called the "power" of the test.

A criterion that appears convincing is, to keep the probability of error of first kind or type I less than or equal to a prescribed value and minimise the probability of error of second kind or type II

A famous theorem of Neyman and Pearson[10] states that a likelihood ratio test will satisfy this criterion. Thus, in terms of and this amounts to minimizing (in radar, maximization of probability of detection) for a given predetermined value of (false alarm probability). This is also called as the Neyman-Pearson criterion.

What is important to notice is that on final analysis all these criteria lead to the same generic form of test.[8] As mentioned before, if $P_o(X)$ is the probability density function that if H_o is true the observed waveform x(t) could have arisen; and $P_1(X)$ is the probability density function that if H_o is false or if H_1 is true, x (t) could have arisen. Then the two possibilities given by the test are,

1. reject H_o (or accept H_1) if $P_1(X)/P_o(X) > \lambda$
2. accept H_o if $P_1(X)/P_o(X) \leqslant \lambda$

for a λ that is a real, nonnegative number the critical region $X_1(\lambda)$ which consists of all X for which $P_1(X)/P_o(X) > \lambda$ gives a test of H_o against H_1, of maximum power, from among all tests of level $P_o(X_1(\lambda))$. λ depends upon a priori probability and costs if these are known (Bayes' test), or on the predetermined value of α (Neyman - Pearson test). What is important to notice is that it is not dependent on observation of x (t). What the test means is, we are to examine the possible causes of what we have observed and to determine whether or not the observation is λ times "more likely" to have occurred if H_1 is true than if H_o is true; if it is, we accept H_1 as true and if not we

accept H_o. If $\lambda = 1$, it is the cause that is chosen which is more likely to have given rise to x (t). $\lambda > 1$ indicates a bias on the part of the observer to choose one hypothesis over another.

Now it is possible to arrive at the proper mathematical expression for MF by making the following assumptions:

1. noise, n (t), is additive, gaussian and white with spectral density $N_o/2$.

2. Signal, if and when present, has the known form s $(t - t_o)$, $t \le t_o \leqslant t \leqslant t + T$ where the delay t_o and signal duration T are known.

3. Signal x (t) is observed in some observation interval I, which includes the time interval specified in 2.

Now, the hypotheses to be tested are,

H_o ; x (t) = n (t) t in interval I

H_1 ; x (t) = s $(t - t_o)$ + n (t), t in some interval I.

But the probability density of a sample n (t) of white gaussian noise lasting from a to b, can be expressed as

$$p(n) = k \exp \{ - \frac{1}{N_o} \int_a^b n^2(t)\, dt \} \qquad (12)$$

where $N_o/2$ is as usual the double-ended spectral density of noise, k is a normalising constant of the distribution not dependent upon n (t). Hence,

$$p_o(x) = k \exp \{ - \frac{1}{N_o} \int_I x^2(t)\, dt \} \qquad (13)$$

is simply the probability density that noise waveform can assume the form of x (t) and that H_o is true. Similarly if H_o is false or H_1 is true,

$$p_1(x) = k \exp\{-\frac{1}{N_o}\int_I (x(t) - s(t-t_o))^2\, dt\} \quad (14)$$

is the probability density that the noise can assume the form $n(t) = x(t) - s(t-t_o)$, if H_1 is true.

$P_1(X)$ can be expanded thus:

$$= k \exp\{-\frac{1}{N_o}\int_I x^2(t)\, dt + \frac{2}{N_o}\int_I x(t)\, s(t-t_o)\, dt$$

$$- \frac{1}{N_o}\int_I s^2(t-t_o)\, dt\} \quad (15)$$

But $\int_I s^2(t-t_o)\, dt$ is equal to E, the energy of the signal.

On substituting (13) and (14) in ( ), we have,

$$= \frac{p_1(x)}{p_o(x)} = \exp\{\frac{2}{N_o}\int x(t)\, s(t-t_o)\, dt - \frac{E}{N_o}\} \underset{\leq\lambda}{>\lambda \text{ or}}$$

Denoting $\int_I s(t-t_o)\, x(t)\, dt = y(t_o)$ and taking logarithms on both sides (15) becomes

$$\frac{2\, y(t_o)}{N_o} - \frac{E}{N_o} \quad \begin{matrix} > \\ \text{or} \\ \leq \end{matrix} \quad \log \lambda \quad (16)$$

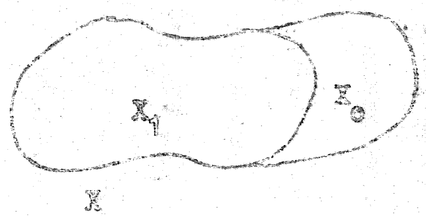
Rearranging this,

$$y(t_o) \begin{matrix} > \\ \text{or} \\ \leq \end{matrix} \frac{N_o}{2}\log \;\; + \frac{E}{2} \quad (= \lambda') \quad (17)$$

Changing variables in (15) A by setting $t_o - t = \tau$ we get,

$$y(t_o) = \int_{-T}^{0} s(-\tau)\, x(t_o - \tau)\, d\tau \quad (18)$$

But this is nothing but the output, at time $t = t_o$, of a filter



X = sample space

X_0 = acceptance region (accept H_0)

X_1 = rejection region (reject H_0)

Fig. 2. (b) : Illustrating acceptance and rejection regions.

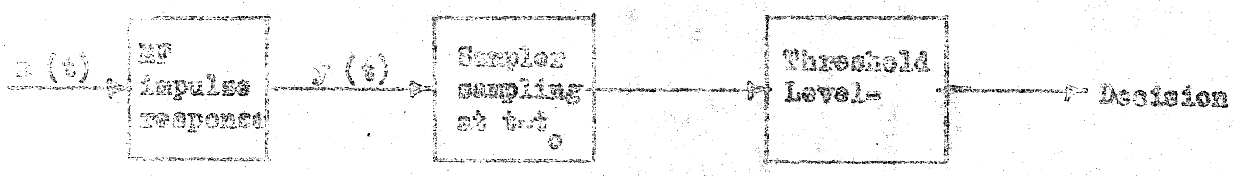


Fig. 2. (c): A simple radar detection system.

of a filter with impulse response $g(\tau) = s(-\tau)$ to an input wave form $x(t)$. Thus, this is clearly matched to $s(t)$ and the optimum radar detection system takes the following form, as in Figure 3.

The equation (18) needs an explanation as it may be interpreted in two different ways. One way is to note that $x(-\tau)$, and hence $g(\tau)$, is not identically equal to zero, only in the interval $-T \leqslant \tau \leqslant 0$ and hence this justified the limits of integration in (18). However, in order to realize $g(\tau)$, a delay of $\Delta \geqslant T$ must be inserted in $g(\tau)$ as is also required by the definition of MF. But as this introduces an equal delay at the filter output it must then be sampled at time $(t_o + \Delta)$ rather than at t_o. This leads to MF detection. Alternatively $y(t_o)$ could also be obtained by implementing equation (18) literally, as follows: Multiply $x(t)$, the incoming waveform, by a stored replica of the signal waveform $s(t)$, delayed by t_o, compare the product $y(t_o)$, integrated over the observation interval, with the threshold λ'. This leads to correlation detection.

The solutions represented in Figs. 1 and 3 can be seen to be essentially the same and the MF of Fig. 3 in fact maximises the SNR of $y(t)$ at $t = t_o$. Thus the two criteria namely the mean square and probabilistic criteria lead to the same result in case of additive white gaussian noise. However, it may be mentioned, the specification that noise should be additive is not as rigid in the second case, as it is in the first.

3. <u>Properties of Matched Filters:</u>

The Figure 3(a) illustrates a schematic, signal generator and MF pair. It operates as follows:

A unit impulse (at time $t = 0$), excites a filter G_1, with an impulse response $s(\tau)$, to produce a signal $s(t)$. To this, a white noise of waveform $n(t)$ (of power density $N_o/2$), is added. The combined signal $x(t)$ is passed through another linear filter G_2, matched to $s(t)$. Let $y(t)$ be the output of G_2. As the whole process is linear, output signal may be resolved into two components,

$$y(t) = y_s(t) + y_n(t) \qquad 3.1.1.$$

$y_s(t)$ is that portion of $y(t)$ due to $s(t)$ as if it acted alone and similarly $y_n(t)$ is that component due to $n(t)$ only.

The response of a linear filter with an impulse response $h(\tau)$, to an input $s(t)$ is,

$$\int_{-\infty}^{+\infty} h(\tau)\, s(t-\tau)\, d\tau$$

when $h(\tau) = s(-\tau)$ as in the present case

$$y_s(t) = \int_{-\infty}^{+\infty} s(-\tau)\, s(t-\tau)\, d\tau$$

This can be seen to be symmetric in 't', since,

$$y_s(-t) = \int_{-\infty}^{+\infty} s(-\tau)\, s(-t-\tau)\, d\tau$$
$$= \int_{-\infty}^{+\infty} s(t-\tau')\, s(-\tau')\, d\tau' = y_s(t)$$

Putting, $\tau' = t + \tau$.

$y_s(t)$ may look like the pulse in Figure 3 (b)

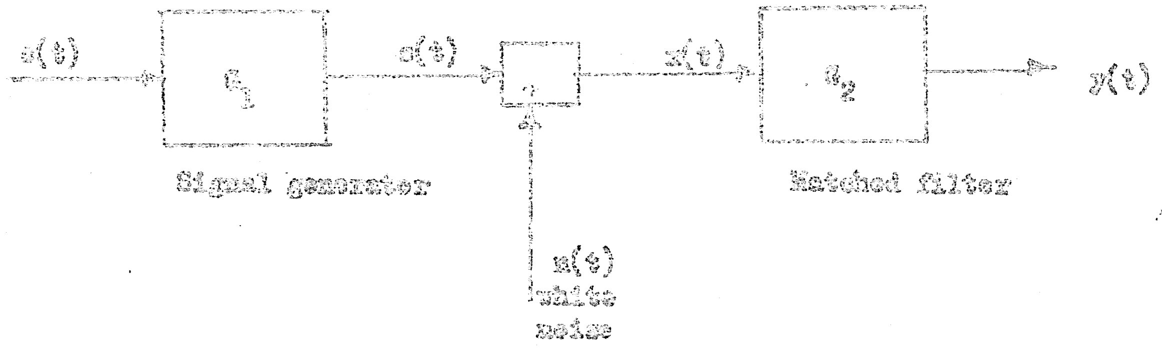


Fig. 3(a) Illustrating MF operation.

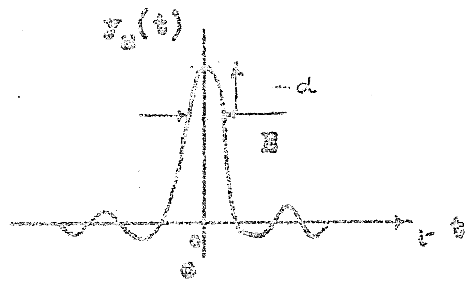


Fig. 3 (b)

The height of the pulse at the origin is,

$$y_s(0) = \int_{-\infty}^{+\infty} s^2(\tau)\, d\tau = E$$

E being signal energy, $y_s(t)$ cannot exceed $y_o(0)$ for any 't' as the application of Schwarz's inequality (8) has already shown. Thus it can be seen that the signal has some finite width and height and is not actually an impulse. But because the signal component of y (t) has to be made as large as possible at some instant of time, to be sure of the presence of the signal, the output signal component should ideally be an impulse rather than the finite height, non zero width MF output of Figure 3 (b). An inverse filter with an impulse response $1/s(j\omega)$, instead of MF is required to obtain this impulse. Because, then the output signal component would then be the impulse response of the cascade of generator and inverse filter which is nothing but unity, resulting in an impulse.

But however, this cannot be used because any physical signal must have a spectrum $s(j\omega)$ which approaches zero for large 'f'. But it is exactly the reverse in case of an inverse filter as it becomes indefinitely large as f tends to ∞. In fact output noise component smothers the output signal component because the power in the output noise component of inverse filter will be infinite as the input noise is assumed to extend over all frequencies.

Thus, a compromise has to be effected. That is, to keep output signal component as close to an impulse as possible, at the same time, to suppress the noise outside the signal band as effectively

as possible. This is done by MF, thus;

Writing a signal spectrum in the form

$$S(j\omega) = |S(j\omega)|\, e^{-j\phi(f)}$$

where $\phi(f)$ is phase spectrum of $s(t)$.

But inverse filter hasthe form,

$$\frac{1}{S(j\omega)} = \frac{1}{|S(j\omega)|}\, e^{j\phi(f)}.$$

However, noise suppression cannot be achieved by modifying the phase characteristics of inverse filter. Because noise at any frequency has random phase. So, the amplitude characteristic that is needed, is the one that is small when signal is small compared to noise, and large when signal is large relative to noise. That is nothing but $s(j\omega)$.

Hence, we want such a filter whose impulse response is,

$$H(j\omega) = |S(j\omega)|\, e^{j\phi(f)} = S^{*}(j\omega)$$

This, as found earlier, maximises SNR and is the best compromise possible.

3.1. <u>ρ_0 The Figure of Merit:</u>

The dimensionless quantity - $2E/N_0$ is a fundamental one and it is the most important parameter in the calculation of the performance of systems using MFs. It is to be noted, that this single parameter describes many things: For example, ρ is SNR in $y(t)$, it is also the peak value of output of MF and it does not depend upon the signal wave-form.

As already explained before, for a signal

$$x(t) = s(t) + n(t)$$

there is a $y(t) = y_s(t) + y_n(t)$.

This $y(t)$ is a filtered form of $x(t)$ and it maximises SNR, but has the peculiarity that larger the input signal to noise ratio in $x(t)$, the large is the noise component in y and of course the signal is still larger. (The mathematical consequences of this feature are highly significant and are discussed elsewhere.[11]). This is best revealed in the following relation that is given, between output SNR ρ_o and input SNR ρ_i :

Let B_N represent the bandwidth of a rectangular band filter, with the same maximum gain, having the same output noise power as MF. N_{in} represents the amount of input noise power within MF band and is put as

$$N_{in} = B_N N_o$$

Further let average signal power at filter input be

$$P_{in} = E/T; \quad T = \text{effective duration of signal,}$$

Putting $\rho_i = P_{in}/N_{in}$,

$$\rho_o \text{ becomes } = 2 P_{in} T/N_o$$

$$= 2 T_i N_{in}/N_o = 2 T \rho_i B_N$$

Therefore, $\rho_o = 2 B_N T \rho_i$

Hence, MF brings out a gain of $2 B_N T$ in SNR at its output. This means

the input average signal power P_{in} can be controlled at will, by lengthening T, the duration of the signal, for a given total signal energy, because $P_{in} T = E$ and ρ_i can be small in that case. Thus, all the increase in ratio ρ_o to ρ_i that is obtained, is caused by "spreading out" the fixed energy signal, is offset by a pr proportionate decrease in ρ_i. Alternatively if an increase in ratio of ρ_o to ρ_i is affected by increasing signal bandwidth, it may be inferred b that this is due to decrease of ρ_i. A large bandwidth, i.e. a large B_N implies that N_{in} has to be large. Consequently the amount of input noise taken in through the increased MF bandwidth is also larger. However, when the interfering noise is band limited, white and of fixed total power, then ρ_i is no longer dependent on B_N, because, total power N_{in} will spread out evenly, over the whole signal "band" irrespective of its width B_N. It can be clearly seen, that in this case, out of all signals, of equal energy, that one with the largest bandwidth is the most suitable signal. The value of N_o is smallest in this case.

3.2. Pulse Coding:

The functions $s(j\omega)$ and $s^*(j\omega)$, require careful study from the point of view of "coding"a and "decoding". Although the impulse at the input to the signal generating filter has all frequency components, their amplitudes and phases are such, that they reinforced each other only at $t = 0$ and not any where else. What the transfer function $s(j\omega)$ does, is to "code" the amplitudes and phases of all these frequency components, so that the input pulse is stretched

to an arbitrary waveform of time duration say from $t = 0$ to T. Now the receiver, that is MF, with transfer function $s^*(j\omega)$ has to decode the signal i.e. it should restore all the amplitudes and phases to their original values. Here, a compromise has to be effected, as it is not possible to restore both without resorting to an inverse filter. Hence only phases are restored (by the process of auto-correlation), so that all frequency components at the filter output have zero phase at say $t = 0$ and reinforce to give a high pulse, with a width not less than the order of the reciprocal of signal bandwidth.

Thus, what actually "coding" does, is to spread the signal energy, over a duration of time T and in "decoding", the whole energy of the signal is collapsed into a pulse approximately T times narrower, than the input pulse where is some suitable measure of signal bandwidth. Such a collapsing of energy enhances SNR by a factor $B_N T$ times, as already mentioned.

The idea of coding is illustrated[12] in the Figs. 3.2(a), (b) and (c). Figure 3.2. (a) shows a pulse s (t) of 1-volt amplitude and 1 second duration, appearing across 1 ohm load resistor (a rectangular pulse is used here because, it contains maximum energy when generator is peak power limited.)

Energy in $s(t)$ = 1 watt sec. = 1 joule.
MF for this $s(t)$ has an impulse response $h_1(t) = s(-t + 1)$, Figure 3.2. (b).

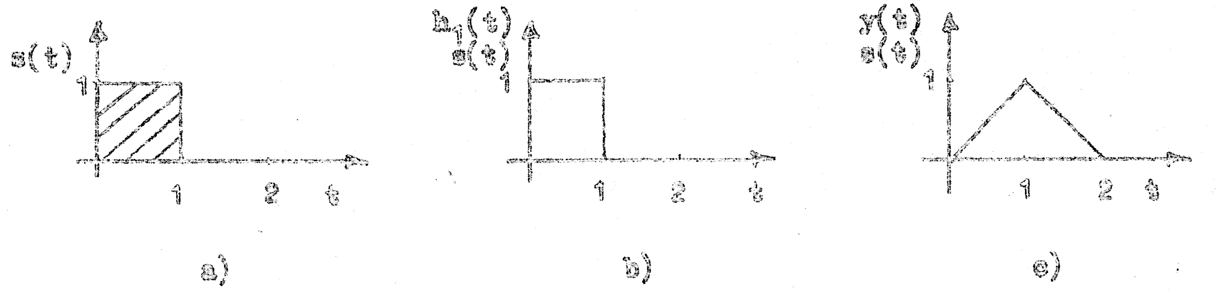


Figure 3.2. Example of MF, a) Input Signal, b) Impulse Response, c) Output Signal.

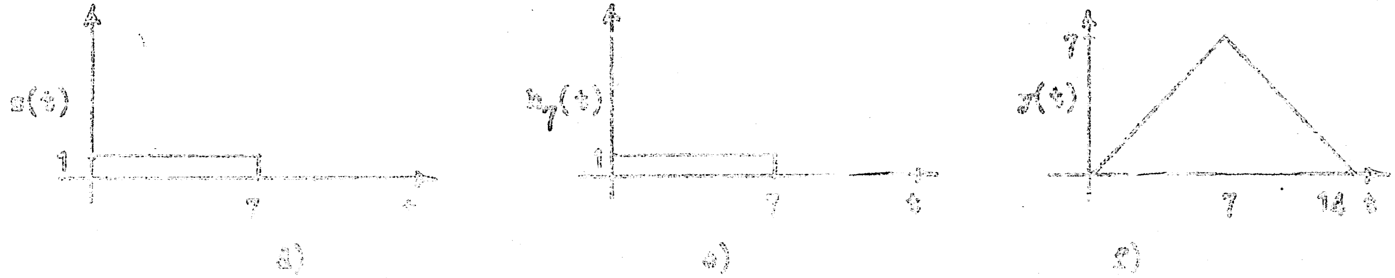


Figure 3.2. Effect of Increasing Signal Energy, a) Input signal, b) Impulse Response
c) Output Signal.

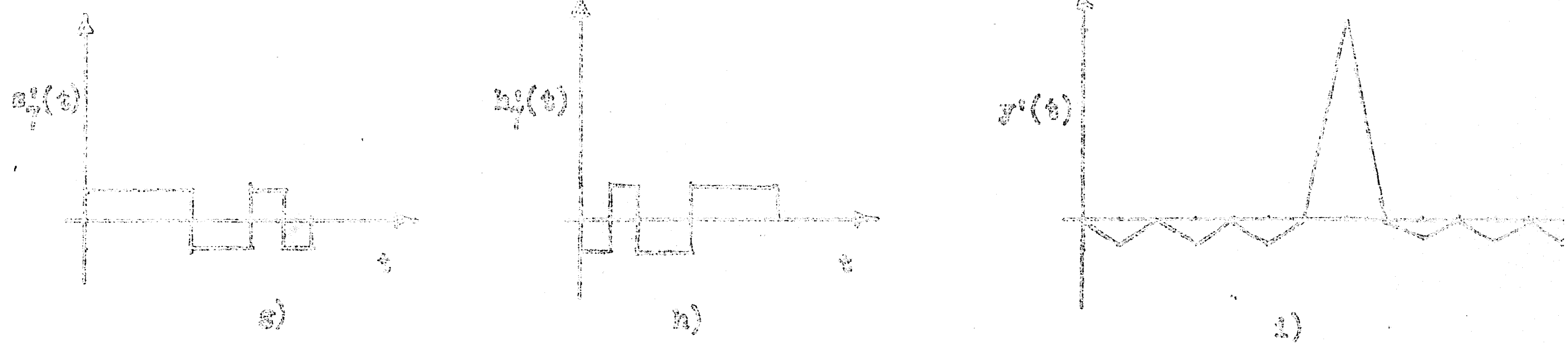


Figure 3.2. Effect of Coding the Signal, a) Coded Input Signal, b) Impulse Response,
c) Output Signal.

$$\text{Output } x(t) = \text{Convolution of } s_1(t) \text{ with } h_1(t)$$

$$= \int_{-\infty}^{+\infty} s(\tau)\, h_1(t-\tau)\, d\tau . \qquad \text{Figure 3.2.(c)}$$

If energy in $s_1(t)$ is increased to 7 joules by lengthening of the pulse to 7 sec. duration, the peak output voltage is also increased by a factor of 7, so also the duration of the signal becomes doubled, as illustrated in Figs. 3.2. (d), (e) and (f). In this case when noise is present, it is very difficult to locate the peak as to when actually it has occurred, i.e. resolution of the signal is poor. Hence pulse coding is employed to change the shape of s(t) so that its energy may be increased while keeping the resolution to a desired degree. This is very well illustrated in Figs. 3.2.(g) (h) and (i). Peak of output signal, in Fig. 3.2.(i) is the same as in Fig. 3.2.(f) but the width is the same as in Fig. 3.2.(c) for the peak.

Thus by coding of the pulse, what has been done is just a manipulation of $s(j\omega)$ to obtain a desired resolution.

Tapped delay lines provide one of the best methods of generating these matched signals and such a MF is discussed in the next section.

—

4. Tapped delay line Matched Filters:

4.1. Sampling Theorem:[13]

This theorem states, that if a function $f(t)$ contains no frequencies higher than W cps, it is completely determined by giving its ordinates at a series of points, spaced at 1/2W sec. apart.

This implies that a channel, having a bandwidth W, which is available for use for a length of time T can transmit signals without loss of information provided, the signals are of finite bandwidth W and of finite time duration T. Although it is not possible to fulfil both of these conditions exactly, it is found adequate in many cases, if the spectrum can be within the band W and the signal amplitudes are "negligibly" small outside the interval T.

Proof of this theorem is as follows:

Let $F(j\omega)$ represent the spectrum of a function $f(t)$,

then,

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-2\pi W}^{+2\pi W} F(j\omega)\, e^{j\omega t}\, d\omega$$

because $F(j\omega)$ is assumed to be equal to 0, outside the band W. Putting $t = n/2W$, n being positive or negative integer, $f(t)$ becomes,

$$f(n/2W) = \frac{1}{2\pi}\int_{-2\pi W}^{+2\pi W} F(j\omega)\, e^{j\omega(n/2W)}\, d\omega$$

LHS of the equation gives the valuesof $f(t)$ at the required sampling points. The integral on RHS, is the nth coefficient in Fourier series expansion of $F(j\omega)$, in the interval from $-W$ to $+W$. This clearly means, $F(j\omega)$ is completely determined because, the values of the samples $f(n/2W)$ determined the Fourier coefficients, in the series expansion of $F(j\omega)$. As the spectrum of the function is completely known, its time function can also be known by taking its inverse Fourier transform. Hence, original samples determined $f(t)$ completely. In fact, there can be one and only one function whose spectrum is limited to a bandwidth W and which posses through the given values at sample points separated 1/2W secs. apart.

Mathematical representation of $f(t)$ is,

$$= \sum_{n=-\infty}^{+\infty} x_n \left\{ \frac{\sin \pi (2Wt - n)}{\pi (2Wt - n)} \right\}$$

At every sample point a 'pulse' of $(\sin x/x)$ form is addedand its amplitude is adjusted to equal that of the sampb $f(t)$ representing summation of such elementary pulses.

One of the most important conclusions of sampling theorem, and which favors the construction of a MF using tapped delay lines is the followings If the function is limited to the time interval T and the samples are spaced 1/2W secs. apart, then the 2TW samples taken over the interval, describe the original function completely, and no other samples need be considered. It is evident from the previous section, that the time bandwidth product of the signal or

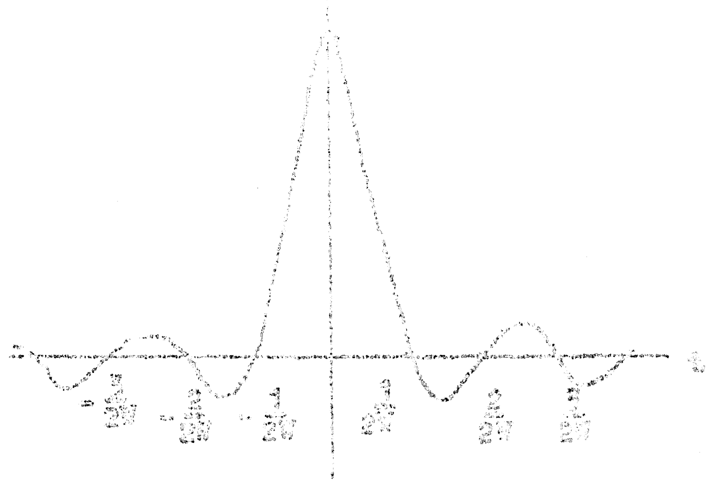


Fig. 4 (a) $\Delta t = 1/2W$

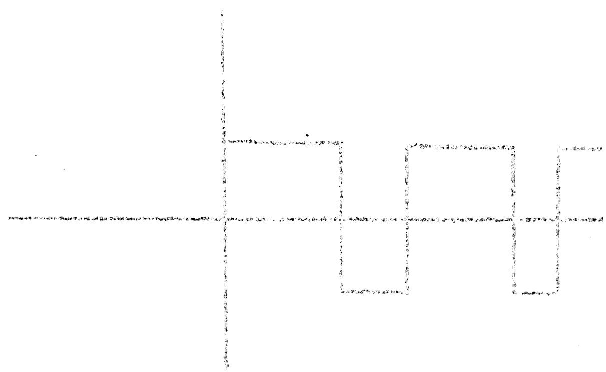


Fig. 4 (c)

its MF is an important parameter in describing the filter. It should be noted that a time bandwidth product is equal to the improvement factor for SNR, as defined in the previous section, at the output of the filter. Further more, the complexity of the filter, is also given by the TW product, as proved by sampling theorem.

4.2. Application of Sampling Theorem for MF:

Let the time inverse of the matched signal be called $\sigma(t)$. If $\sigma(t)$ has a bandwidth of W cps and a time duration of T secs. then, by sampling theorem, it is seen that it may be completely determined by 2TW amplitude samples, taken between t = 0 and t = T. If the 2TW samples are regularly spaced in time, $\sigma(t)$ is given by

$$\sigma(t) = \sum_{i=1}^{2TW} a_i \frac{\sin \pi (2Wt - i)}{\pi (2Wt - i)}$$

as shown earlier, where a_i is the sample value at the i th sample position.

Similarly $a_1 = \sigma(1/2W)$

$a_2 = \sigma(2/2W)$ and so on.

$$\frac{\sin \pi (2Wt - i)}{\pi (2Wt - i)}$$

is the sampling function shown in Fig. 4 (a).

Hence, to synthesize a matched filter, we have to build a filter such that for an impulse input, its output is a series of sampling function of proper amplitude and position so that the equation (   ) is satisfied.

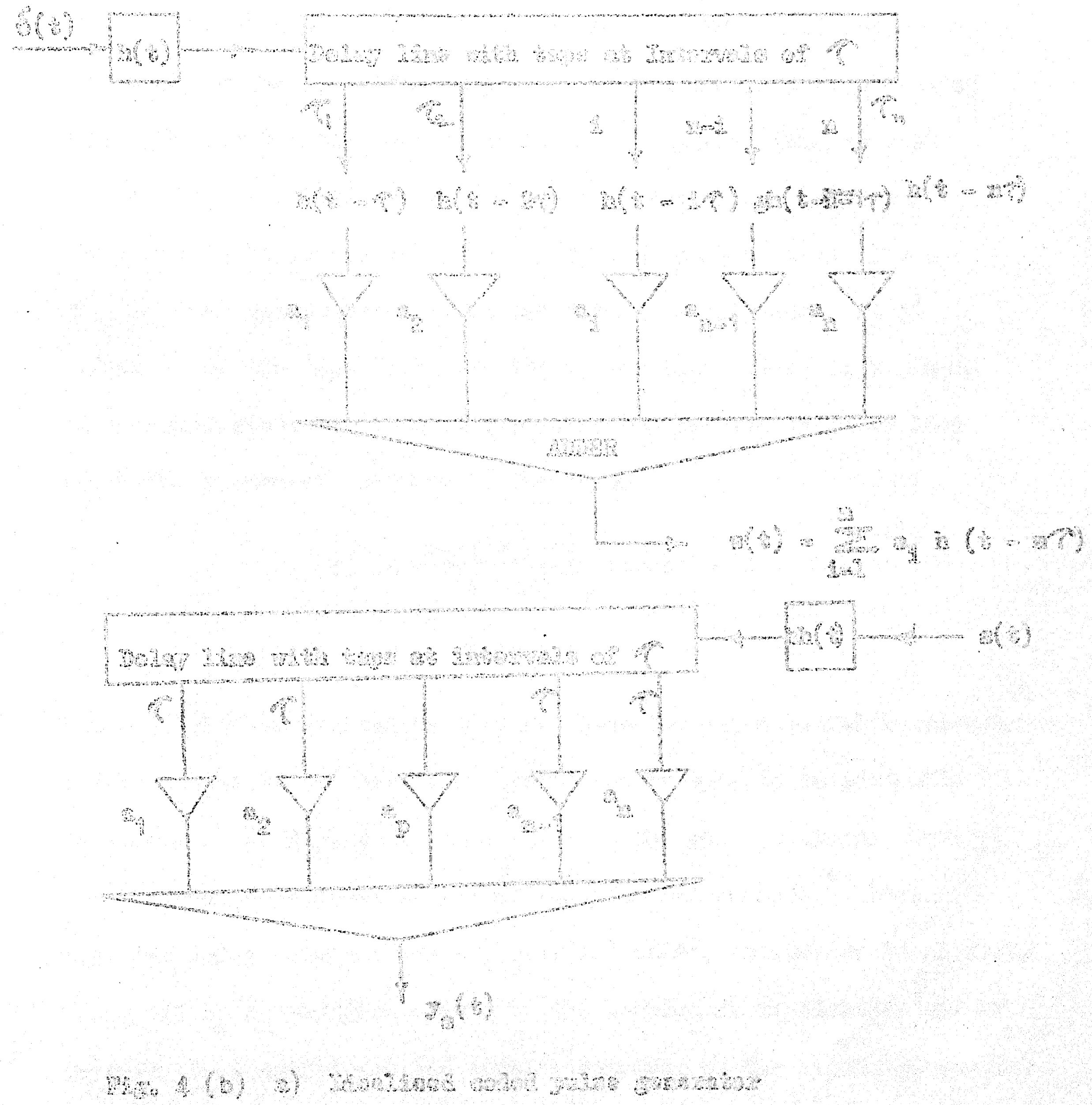


Fig. 4 (b) a) Idealised coded pulse generator

b) The Matched Filter.

An ideal low pass filter of bandwidth W has an impulse response which is one of the desired sampling functions. Therefore, the ideal low pass filter, followed by a device which will repeat the low pass filter output with proper position and amplitude 2TW times, will be the required matched filter. A tapped delay line, with associated circuits as shown in Figure 4 (b) satisfies the requirements of such a device. However, an ideal tapped delay line, is almost impossible to construct. Further, the low pass filter that is needed is not physically realizable. However, these difficulties are overcome in the case of practical MFs by the low pass characteristics of the actual delay lines. In an actual MF of the type described above, low pass filter is replaced by a signal generator to feed a practically realisable delay line which has a impulse response of the form,

$$h(t) = \frac{\mathrm{Sin}\,(x - \tau)}{(x - \tau)} \quad \text{for } t \geqslant 0$$

$$= 0 \quad \text{for} \quad t < 0$$

Which is what we desired to sample. Here the input signal is clearly not band limited and hence the system can be seen to be perfectly realizable. In Fig. 4 (b), the a_i's or the gain functions have been chosen as + equal to + 1 or -1*, for convenience.[8] Now, with the input looking like a (Sin x /x) pulse, through an ideal delay line, and a_i's assigned + or - 1, the problem of realization of MF is completely solved and the impulse response of the signal generating filter, has the form of a low pass sequence of positive and negative pulses as shown in Fig. 4 (c).

The problem of searching for a desirable signal, reduces to that of finding a desirable sequence, $a_o, a_1, \ldots a_n$, of + 1's and - 1's. Referring to Fig. 3.2. (b), it will be found that this is a type of signal that is needed. It can also be seen that s(t) , is a constant amplitude signal which is a highly desirable property, of all radar and communication signals, arising from the use of a peak-power limited trahsmitter. For such a transmitter, the operation at rated average power, often demands the use of amsconstant amplitude signal i.e., one in which only the phase or the zero crossing of pulses is modulated.

There are several advantages, of having a single filter that can generate as well as process a signal. The important ones being, their ease in construction, suitability for radar where transmitter and receiver are physically at the same location.

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5. Signal Selection Problem:

5.1. Signal desirability:

As found in the last section, the choice of a signal and its MF or a set of signals and their MFs, is limited only by, bandwidth and time duration considerations. However, while the choice for a certain bandwidth, is generally dictated by the system as a whole, the signal duration, is partly limited by the physical size and complexity of the MF one can build. Thus the length of delay line, the number of taps provided and hence the general complexity of the filter are determined by the length of the matched signal. ~~For-a~~

For a given bandwidth and time duration, there are, many possible signals, one can think of. The problem then, is to choose the "most desirable" signal. This is defined as the signal which when passed through its MF, yields high narrow pulse with as low a clutter as possible, on either sides of the peak. It is possible to discover such signals, by making a careful study of their MF outputs, as an MF output is nothing but the auto correlation function of the signal itself.

The output of the MF is,

$$y_s(t) = \int_{-\infty}^{+\infty} s(\tau)\, s(\tau - t)\, d\tau$$

which is the auto correlation function of $s(t)$, as mentioned before, having the following properties:

1) $y_s(-t) = y_s(t)$;
2) If $s(t)$ has a duration T, then the duration of $y_s(t)$ is 2 T.

3) It has a maximum at $t = 0$

maximum $y_s(t) = y_s(0) = \int_{-\infty}^{+\infty} s^2(\tau)d\tau$ and is proportional to E the total energy of the signal.

Thus $y_s(t)$ of same maximum value may be obtained either from a short duration signal with high average power or a long duration signal with low average power. Peak power limitations often dictate the use of latter signals, but this will increase the complexity of the filter, as a long signal is needed, in this case.

5.2. Clutter studies of certain auto correlation functions:

In the figures 5 (a), (b) and (c), some signals and their auto correlation functions are shown which reveal certain important characteristics. In all these figures, it can be seen that there is a pulse like central peak, with minor peaks or clutter placed symmetrically, on either side. A comparison between the figures 5. (b) and 5.(c) clearly reveals, the effect of clutter, on the main central peak. In these figures, two signals of equal length and total energy, and their auto correlation functions are considered. The difference in the nature of clutter in both cases is quite plain. In the figure 5 (b), the clutter or the numerous minor peaks are powerful and too close to the main peak so that the detection of the main peak, in the presence of noise, becomes difficult. Because, noise amplitude can add or subtract from the auto correlation function peaks and bring out number of peaks having same amplitudes, so that it becomes difficult to determine which peak is due to the main signal. This problem is more serious, when there is

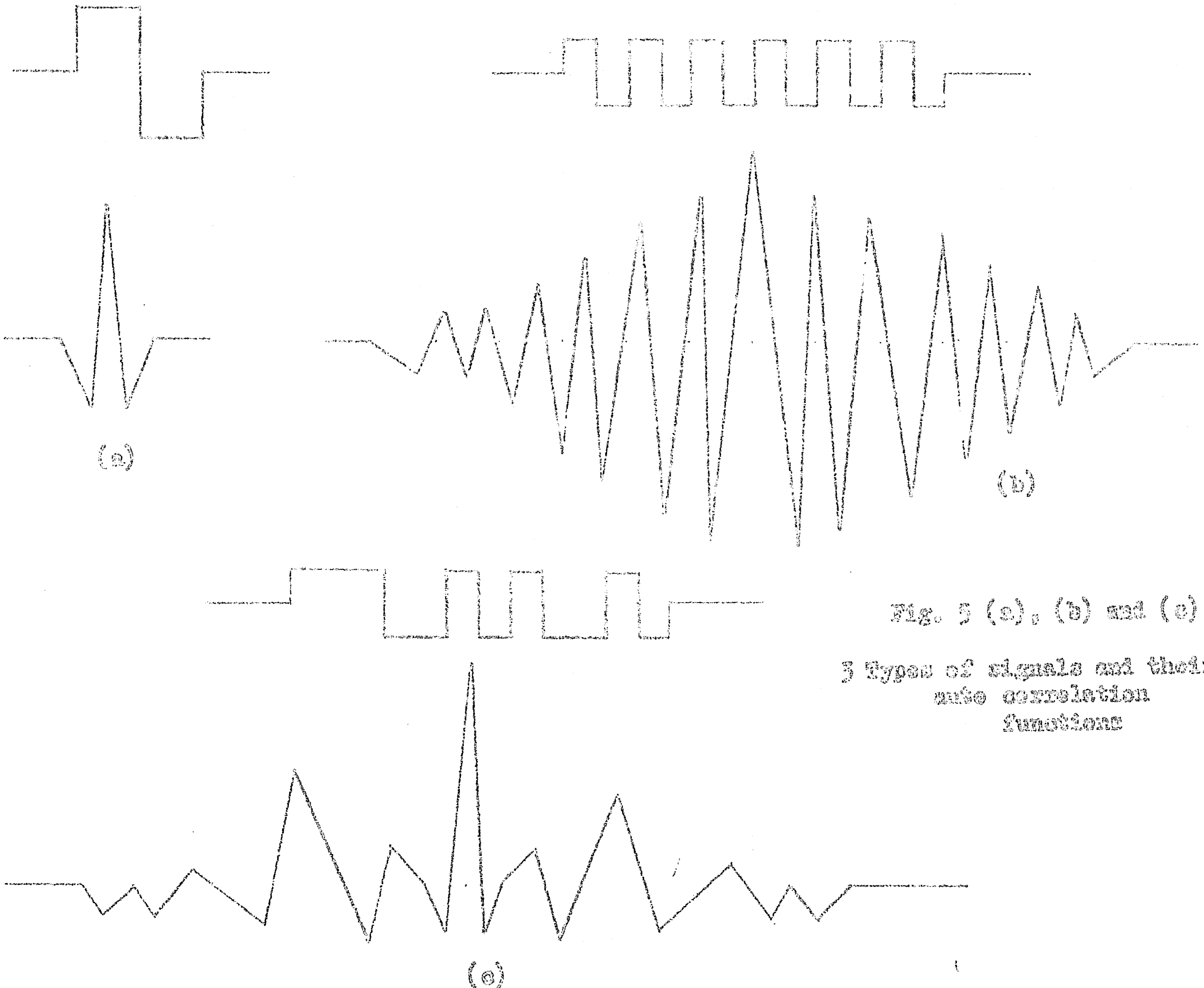

Fig. 5 (a), (b) and (c)

3 Types of signals and their auto correlation functions

an unknown delay in the signal itself, due to moving targets. In Figure 5 (c), however, the clutter is well spread and this comparison illustrates the fact that the signal should be as 'noise like' as possible and certainly not periodic as in Figure 5 (b). By 'noise like' it is meant that the nature of occurrence of + 1's and - 1's is without any periodicity. The more complex or 'noise like' the signal is, the less is the clutter on either sides of the main peak. Another property of the noise like signal, is the fact, that such a signal generally has a frequency spectrum, in which almost all frequencies are equally weighted. On the contrary, a signal having 'well-ordered' + 1's and -1's as found in the signal in Figure 5 (b), has a spectrum in which some frequencies are more dominant than others. In other words, a noise like signal utilises its bandwidth to a greater extent, than a less complex signal. Thus, the bandwidth of a signal gives an idea of the degree of its noiseness. A measure of the effective bandwidth of a signal that is often used, is,

$$W_{eff} = \frac{\left[\int_0^\infty |s(\omega)|^2 \, df \right]^2}{\int_0^\infty |s(\omega)|^4 \, df}$$

It may be noted that the advantages of a long noise like signal, are two fold. Firstly, since the amplitude of the central pulse is proportional to the total energy in the matched signal, the energy in the transmitted signal may be spread over a comparatively long time interval when using a long noise signal rather than when using a narrow pulse, uncoded, matched signal, concentrated over a short-time, given that the total energy of the signal is the same in both the cases. Secondly since number of possible noise like signal

increases with duration, long signals can be used to transmit more messages than shorter ones. Also, a filter matched to a certain noise like signal produces a noise like response to another noise like signal. This means by coding the signal, the advantage to be gained is also secrecy. Because, a noise like matched signal is detectable as a pulse only to some one possessing a filter matched to that particular signal. For others, it does not convey any information and appears as noise.

5.3. Mathematical Criteria for Signal Selection:

Let the matched signal s(t) consist of, a set of (N (N=2TW) sample values, a_1, a_2, ... a_N. The auto correlation function of s (t) may then be considered to consist of a set of (2N - 1) sample values, $P_{-(N-1)}$, $P_{-(N-2)}$, ... P_{-2}, P_{-1}, P_o, P_1, P_2, ... P_{N-1} which are given by, the following equations:

$$P_o = \frac{1}{2W} \sum_{i=1}^{N} a_i^2$$

$$P_1 = P_{-1} = \frac{1}{2W} \sum_{i=1}^{N-1} a_i a_{i+1}$$

$$P_2 = P_{-2} = \frac{1}{2W} \sum_{i=1}^{N-2} a_i a_{i+2}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$P_{N-1} = P_{-(N-1)} = \frac{1}{2W} a_1 a_N$$

These expressions are obtained by solving equations ( ) and ( ) the value of P_o, being the amplitude of the central pulse is of

particular importance. The scheme of tapped delay line filters with amplifiers as gain functions, allows any reasonable constant to be substituted for $1/2W$, in the above equations. Taking it as unity for convenience, we have,

$$P_k = \sum_{i=1}^{N-|k|} a_i \, a_{i+|k|} \qquad (1)$$

$$k = (0, \pm 1, \pm 2, \ldots \pm N-1)$$

For computational ease, it is convenient to put the above equations in a matrix form, as in Figure 5 (d).

$$\begin{array}{c|cccccc}
 & a_1 & \cdots & \cdots & \cdots & \cdots & a_N \\
a_1 & a_1^2 & a_1a_2 & a_1a_3 & a_1a_4 & \cdots & a_1a_N \\
a_2 & a_2a_1 & a_2^2 & a_2a_3 & a_2a_4 & \cdots & a_2a_N \\
\cdot & a_3a_1 & a_3a_2 & a_3^2 & a_3a_4 & \cdots & a_3a_N \\
\cdot & a_4a_1 & a_4a_2 & a_4a_3 & a_4^2 & \cdots & a_4a_N \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\
a_N & a_Na_1 & \cdot & \cdot & \cdot & \cdots & a_N^2
\end{array}$$

The P's are the sums of the elements, in each upper left to lower right diagonal, same as those given by equation 1.

To obtain a high narrow pulse with low clutter, P_o must be set at a large value and P_1, P_2 etc. set at as small values as desirable. This means, we have (2N-1) non linear equations to solve with N unknowns. But in fact, we have only N equations as the auto correlation function is symmetrical about the P_o and the left diagonal P's are equal to their corresponding P's in the right diagonal.

i.e. $P_{-1} = P_1$; $P_{-2} = P_2$ etc..

In spite of this simplification, it is involving to solve these non linear equations, for an arbitrary chosen set of P's . Also, another restriction, placed on a's is that they must be such,that the signal is noise like. If a restriction may be made on the signal that at any time it can either have a + 1 or - 1 amplitude, then, the solutions of the equations is not a problem. In fact, it has been stated previously, there is much advantage to be gained, by employing such a type of signal. These signals are termed as 'binary signals'. They not only simplify the mathematical aspect of the problem, but also it is easier to generate them by shift register techniques. [13] However, by limiting the amplitudes to + 1 or - 1, the extent to which one can realise a noise like signal is decreased considerably. But this disadvantage is offset by many advantages this scheme has. It is possible to make the matched signal look noise like by choosing successive binary samples in such a way that they do not have any apparant order.

5.4. Criteria for Clutter Definition:

Supposing, a sequence of binary sample values, a_1, a_2, ... a_N, is chosen as the most desirable one. Then,

(1) it should have the least cross correlation with any other cyclic permutation of the same sequence of + 1's and - 1's.

(2) its auto correlation function should have the maximum ratio of central peak amplitude to the average clutter power, when compared to other cyclic permutations of the same sequence.

Although the first condition, is very important from detection and secrecy point of view, it is difficult to meet that condition completely. In fact, in connection with this, there has been an evolution of an extensive Coding theory[13]. But as regards the second condition,the mathematics involved is fairly simple.

A measure of clutter is the ratio of the peak output signal power to the average a.c. power of the clutter. In terms of sample values, this is the ratio of the central pulse sample value squared, to, the "mean squared deviation", of the clutter region sample values about their means.

The best possible sequence of + 1's and - 1's for a length of N sample value, is found out by finding out the ratios ρ_1, ρ_2 etc, for each cyclic permutation and picking out that sequence which gives maximum ρ . As, for a sequence of fixed length N, the amplitude of the central peak is the same irrespective of any cyclic permutation, for maximization of ρ , we need to consider only the clutter region power. In fact, only when the later is minimum, we have ρ_{max}.

Representing, mean squared deviation of the auto correlation function clutter-region sample values about their means, by μ , we have,

$$\mu = \frac{1}{N-1} \sum_{k=1}^{N-1} (P_k - \tilde{P}_k)^2$$

where $\tilde{P}_k$ is the mean value of clutter region sample values.

$$\tilde{P}_k = \frac{1}{N-1} \sum_{k=1}^{N-1} P_k$$

It must be noted here, that we are considering sample values lying only on one side of the central peak as these are symmetrical. Thus, when

is minimum for a particular sequence, that sequence has the largest peak amplitude to average clutter power ratio.

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6. Pseudo Random Sequences, Their Properties and Generation:

It is seen in the previous section, that a sequence in which +'s and -'s, are arranged in as 'disorderly a manner as possible', is the most useful one as a matched signal. One such sequence satisfying this requirement, is the pseudo random sequence. Before discussing pseudo random sequences, it may be necessary to define and state a criteria, for distinguishing random from pseudo random sequences.

6.1. Criteria for Random and Pseudo Random Sequences:[13]

If a finite sequence of numbers or digits is called 'random', it is only with respect to the manner in which the sequence was generated, rather than on the actual terms appearing in the sequence, as Golomb[13], puts it. For example, in coin-flipping, objects are selected from a "sample space", in an unpredictable manner. But still the selection is, according to some probability distribution and hence selection by coin flipping is a random process. If a sequence of +'s and -'s are so chosen, then it should be termed as a random sequence. However, it is important to note, that the term random does not refer to a posteriori considerations as what the sequence looks like and what its properties are, but refers only to the a priori conditions under which it was generated. For example, if a sequence + + + - - + - is generated by coin flipping process, it is a random sequence. But if it is generated by a deterministic device, such as shift register, it is not random. However, owing to the fact that the wave form obtained from a shift register does meet all the statistical tests to determine

the plausibility of randomness of the waveform it is termed as a pseudo random sequence. It may be noted here that this is only an a posteriori criterion, namely, here we are taking the property of the sequence, to determine its random nature. But this is inadequate for defining a priori randomness. Hence it may be stated specifically that, given a set of tests for the appearance of randomness, it is always possible to device a deterministic process to generate Pseudo-random sequence which passes all the tests.

6.2. Specifications for a Pseudo Random (P N) sequence:

It has been discussed elsewhere, that for a signal to be pseudo-random, the code length N has to be of the form either $4m - 1$ ($m=1,2,3,..$) or $2^p - 1$ ($p = 2,3,4, ...$). The two conditions together form the necessary and sufficient condition for a sequence to be pseudo random. The code lengths N that may be obtained for $N = 4m - 1$ and $N = 2^p - 1$ are found to be the same if and only if number N is what is known as a Mersenne prime.[14] However, best known sequences, are m sequences of the form $N = 2^p - 1$. These are also called as maximum length linear recurring sequences, or maximum length linear shift register sequences. For any N of this form, the number of different signals that may be found to give the desirable output characteristics, as discussed before is given by,

$$t = \frac{\phi(N)}{p}$$

where $\phi(N)$ is the Euler phi function of N. $\phi(N)$ is equal to number of integers less than N which are relatively prime to N. The number m t includes signals which are time reversals of each other but not amplitude inverses.

6.3. Properties of Pseudo Random Sequences:

The signals for each value of p up to 4, are enumerated below. (A + indicates a sample value + 1 while a - indicates a -1 sample value.)

P = 2	N = 3		+ + -
P = 3	N = 7		+ + + - - + -
P = 4	N = 15	+ + + + - - - + - - + + - + -	

The number of signals could be increased by including their time inversions and amplitude inversions. However it may be noted amplitude inverse is not new signal at all, because the response of a filter to the amplitude ofinverse of its matched signal is the amplitude inverse of the auto correlation function itself. (The filter response to the time inverse of its matched signal however appears as noise).

It is interesting to compare the pseudo random signals with signals whose N sample values are determined by fair coin flapping. In the latter case, the probability of any sample value being + 1 is 1/2 and the probability of the same being - 1 is also 1/2. Thus in a random ensemble the average number of +'s and -'s is each = N/2. For pseudo random sequences however, N is always odd being equal to 2^p-1 or 4m - 1 so that the numbers of +'s and -'s cannot be equal even on the average. But it may be easily verified that difference between +'s and -'s is one 1 , for all values of p. Consequently, for large p's, the number of +'s and -'s are almost equal. The signals listed above illustrate this point.

An important feature of these pseudo random signals is the fact that when the pulse repetion frequency of the pulse generator, at input

of matched signal generator is adjusted such that the signal completely overlaps, then, all the clutter on either sides of each peak gets mutually cancelled by the clutter of its adjacent peak to produce a clutter free output and the clutter region sample value sum to - 1. Hence, a PN sequence is also termed as two-level sequence. It may be stated, that all cyclic permutations of a given PN sequence, are good enough to be ~~as~~considered as matched signals to produce completely over lapping MF outputs. But it is different if non overlapping signals are required. Consider the sequence, + + - - + - +, which is a cyclic permutation of the PN sequence ~~xxx~~ + + + - - + - . Its auto correlation function has sample values as follows:

1, 0, -1, 0, -1, -2, 7, -2, -1, 0, -1, 0, 1.

This function is definitely inferior to the auto ~~e~~ correlation function of the sequence + + + - - + - which is,

-1, 0, -1, 0, -1, 0, 7, 0, -1, 0, -1, 0, -1

because, the clutter of the permutated sequence is greater. All cyclic permutations of the original sequence have different auto correlations functions which are inferior to the original and to each other in varying degrees.

To pick out such a PN sequence which has least clutter, is the main problem. It is therefore important to know precisely, the following:

1) For a given length, N the PN sequence, which has the maximum peak to average clutter ratio, unique.

2) which is (are) the sequence(s) of length N having the maximum peak to average clutter ratio, unique.

One of the objectives of the work presented in this thesis, has been to answer those two points.

However, for N = 3, 7, 11, the optimum cyclic permutations are more or less visually obvious as those giving auto correlation functions whose clutter region sample values are 0 and - 1 only. But for higher order N's, auto correlation functions, are extremely complicated. Hence a useful mathematical criteria that is to be used, is the one that is already discussed before, namely, minimising the mean squared deviation of the auto correlation function clutter region sample values about their means.

A computer programme formulated to generate PN sequences of higher order N is discussed in the following sections:

6.4. Generation of PN sequences:

For a code length N, there are 2^N number of sequences possible, where each code has a choice to have a value of + 1 or - 1. To discover such pseudo random signals which are completely different from each other, it is necessary to impose the following restrictions, on the manner in which one has to select them. They are:

1) A PN signal has number of +s and number of -s always differing by 1.

2) Time reversals are to be eliminated. For example + + - is the signal. - + + is its time inverse.

3) Amplitude inverses are also to be eliminated. For example, if + + - is the signal, - - + is its amplitude inverse.

It has been found, these restrictions bring down the possible number of desirable signals from 2^N to 2^{N-3} approximately.

For example, $N = 7$, has $2^4 = 16$ possible signals.

$N = 11$, has $2^8 = 256$ and so on.

Now the next problem is to compute μ for each of these selected signals and pick out that particular sequence which has the least μ. This is the desired optimum sequence. A programme has been prepared on this basis, and it is given in Appendix I.

7. Hardware Realisation of Matched Signal Generator and Filter Pair:

7.1. Introduction:

The requirement is for a 7 code pseudo random signal generator, generating a signal of total duration of time 17.5 uS and its matched filter. Hence time delay / code required,

$$= \frac{17.5}{7} = 2.5 \text{ uS}$$

Therefore, a 7 tap delay line, giving a delay of 2.5 uS per tap and thus a total delay of 17.5 uS, is the one suitable for the above purpose, and two such delay lines are required for the generator and filter.

The 7 code signal referred to above, requires some of the tapped outputs to be inverted (i.e. for an input of + 1 volt the output at those taps have to be -1 volt and vice versa). Hence an inverter is needed for this. In addition, the delay lines that are available for practical use, being non-ideal, attenuate and distort the input pulse at the output from tap to tap. Therefore an amplifier, whose gain is adjustable, is needed, to reshape the pulse at every tap. Finally all the tapped outputs have to be summed up to get the matched signal and hence a summing amplifier is necessary.

7.2. Details of the Delay Lines Used in the Set-up:

7.2.1. Delay line scheme for generator and filter:

Delay lines, each having 10 taps, giving a delay of 0.5 uS per

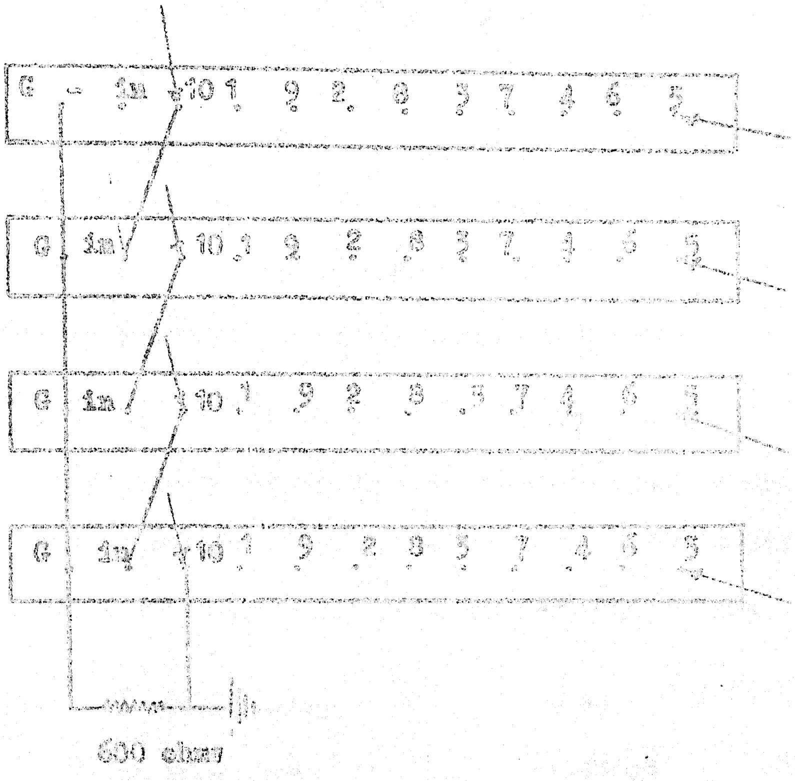

G in 10 1 9 2 8 3 7 4 6 5
G in 10 1 9 2 8 3 7 4 6 5
G in 10 1 9 2 8 3 7 4 6 5
G in 10 1 9 2 8 3 7 4 6 5
600 ohms

Fig. 7.2 (a)

tap are available and hence they are chosen for the above use. As each of the delay lines can only give a total delay of 5 uS, 4 delay lines are required. These are connected in series and 7 taps are drawn out from this arrangement to get the required delay ~~of 2.5~~ of 2.5 uS per tap. This scheme is shown in Fig. 7.2 (a) for generator. The same is repeated for filter also, which means 8 such delay lines are required in all.

7.2.2. Experimental Report on the Available Delay Lines:

Manufacturers of the delay lines are AD-YU Electronics Inc. U.S.A. and their report indicates the following facts:

Total Delay	=	5 uS
Rise time	=	0.45 uS
Characteristic impedance	=	600 ohms
Distortion	=	5 per cent
Attenuation	=	20 per cent

These findings were, for a test pulse of width 6 uS and height 2V for which the output at the 10th tap was a pulse of the same width but the height was 1.6 V resulting in 20% attenuation, as indicated above.

However, the delay lines were tested with a sinusoidal ~~to~~ input to measure the exact attenuation at each tap and these results are given in table 1.

For a Sinusoidal input voltage 2V peak to peak

Tap Nos.	1	2	3	4	5	6	7	8	9	10
Voltage output p/p	1.88	1.82	1.80	1.78	1.75	1.70	1.65	1.60	1.56	1.50

Table 1

% attenuation is thus found to be = 25%

Hence, the design of the amplifier should be such as to compensate for this attenuation.

7.3. Amplifier Design:

7.3.1. Nature of the Amplifier Required; and its Broad Specifications:

As mentioned before, the requirement is for,

1) an amplifier to control the amplitude of code at each tap

2) an inverter to alter the sign of the code at certain taps

and 3) a summing amplifier to sum all the tapped outputs.

It may be easily appreciated that the construction will be simplified to a large extent, if one amplifier is carefully designed to perform all the three operations satisfactorily. Such an amplifier has been designed and constructed and is discussed in the following section.

The broad specifications of the amplifier are:

1) Rise time 0.4 uS

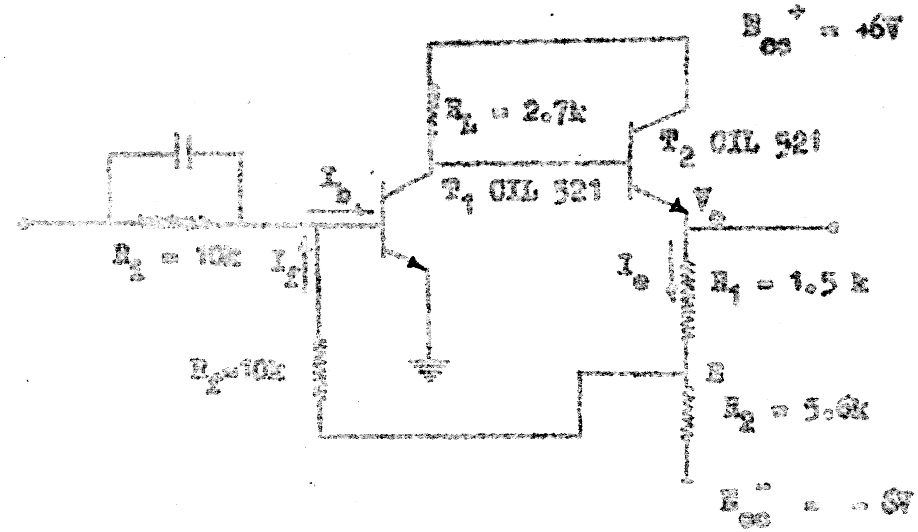
2) Input impedance 10 k

3) Output impedance of the order of 100 ohms

4) gain variable from 1 to 10.

7.3.2. Choice of Configuration:

A common emitter amplifier with high gain which is adjustable by employing a feed back is the obvious choice. This feed back also assures stability for the amplifier. But however, the low output impedance requirement necessitates, the need for an emitter fllower. Hence, the configuration that is chosen, is given in Fig. 7.3.(a).



E_cc^+ = +6V
R_L = 2.7k
T_2 OIL 521
T_1 OIL 321
I_b
V_o
R_s = 10k
I_f
I_e
R_1 = 1.5 k
R_f=10k
B
R_2 = 5.6k
E_cc^- = -6V

Fig. 7.3. (b)

This is tested and seen to meet most of the requirements mentioned above.

R_i is chosen to be = 10 k from input impedance considerations and R_f = 10 k to suit the gain and d.c. bias requirements.

Deciding for an operating point of 1mA and approximately 3 volts, for the first transistor T_1, R_L has to be roughly equal to 3 k. Chosen value is 2.7 k. Taking $_{min}$ of the transistor to be 50, the base current I_b that is needed is

$$= \frac{1000}{50} = 20uA = 0.02 \text{ mA}$$

Hence p.d. across R_f = 10 x 0.02 = 0.2 volt. To this V_{BE} drop is added, so that the point E in Figure 7.3.(a) should have a potential = 0.6 + 0.2 = 0.8 volt.

Again for the second transistor T_2 the operating point chosen is I_c = 1mA and V_{ce} = 3V, so that the point V_e has a potential,

$$= 3 - 0.6 = 2.4 \text{ Volts}$$

$$\text{Hence, } R_1 = \frac{V_e - E}{I_E} = \frac{2.4 - 0.8}{1} = 1.6 \text{ k}$$

Chosen value = 1.5 k.

$$\text{Lastly, } R_2 = \frac{E - E_{cc}}{I_E} = \frac{0.8 - (-6)}{1} = 6.8 \text{ k.}$$

Chosen value = 5.6 k.

Now, the amplifier is checked for its stability. The stability factor 'S' without feed back is = itself, i.e., = 50. So it has to be seen whether the feed back that is provided will bring down SSS to an acceptable value.

Let ΔE, ΔI_f, ΔI_b and ΔI_c be infinitesimal variations from their original values due to variations in I_{co}.

Any change, ΔI_b, in the base current I_b due to I_{co} is correspondingly reflected in I_f and hence ΔI_b can be written as equal to,

$$I_{co} + \Delta I_f$$

This ΔI_b gets β multiplied at the collector, thus bringing out a corresponding change ΔI_c in I_c, the collector current.

Therefore, $\Delta I_c = \Delta I_b + I_{co}$

But ΔI_c is also equal to $S\,I_{co}$, where 'S' is the stability factor, which is to be found out. This is readily obtained by substituting the value of ΔI_b, expressed in terms of the component values namely R_1, R_2, R_L etc. in the above expression for ΔI_c.

The value of S so obtained

$$= \frac{+\,1}{1 + \dfrac{\beta R_L R_2}{R_1 + R_2}}$$

Putting, $R_L = 2.7$ k, $R_2 = 5.6$ k, $R_1 = 1.5$ k $\beta = 50$,

it is found that $S = 51/11.8 \doteq 4.25$.

This is a good and an acceptable value and in fact the feedback has improved the stability approximately by 12 times, which is quite satisfactory for the purpose required.

One another important point to be discussed is the effect of I_{co} variation, due to temperature fluctuations, on the performance of

performance of the amplifier.

In a silicon transistor, I_{co} doubles every $4.5^{\circ}C$[15]. Assuming the worst case to be that when the temperature rise is for example, 30° on Centigrate scale. For CIL 421 transistor I_{co} = 0.1 uA.

Hence I_{co} rises to a value = 12.8 uA for 30° rise in temperature.

Therefore, $I_c = S\, I_{co} = 4.25 \times 12.8 = 56.4$ uA.

Expected value of I_c under this condition,

$$= 1 + 0.0564 = 1.0564 \text{ mA}$$

which is not appreciable enough to chrnge the d.c. biasing drastically. Therefore, the design is satisfactorily safe from this point of view also.

7.3.4. Small Signal Model and Analysis:

The analysis is done as per Fig. 7.3.(b) assuming the emitter follower stage gives a voltage gain of unity and that its effect is only to lower the output impedance.

In Fig. 7.3.(b), for the feed back loop, we have,

$$\frac{V_g - V_i}{R_g} = \frac{V_i}{R_{in}} = \frac{V_i - V_o B}{R_f}$$

Putting $B = R_2/(R_1 + R_2)$ and $V_i = V_o/A$ and rearranging the terms,

$$V_o = \frac{\frac{V_g}{R_g}}{\frac{1}{A}\left(\frac{1}{R_g} + \frac{1}{R_{in}} + \frac{1}{R_f}\right) - \frac{B}{R_f}}$$

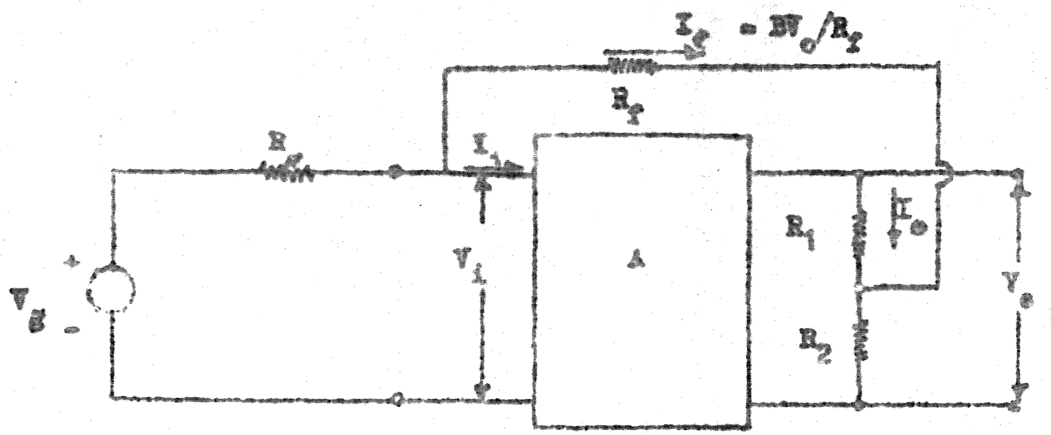


Fig. 7.3. (b)

V_g = generator voltage

V_i = input voltage to amplifier

I_i = input current to amplifier

I_f = current through f.b. resistance R_f

I_o = output current

V_o = output voltage

Current feed back factor

$$= I_f/I_o = \frac{B(R_1+R_2)}{R_f}$$

A = Voltage gain = V_o/V_i

under open loop operation

$$B = \frac{R_2}{R_1 + R_2}$$

Further simplifying this, we get,

$$A_f = \frac{V_o}{V_g} = \frac{R_f}{R_g} \quad \frac{-1}{B - \frac{1}{A}\left(\frac{R_f}{R_g} + \frac{R_f}{R_i} + 1\right)}$$

where A_f = gain with feed back.

As A tends to , A_f tends to $\frac{R_f}{R_g}\ \frac{1}{B}$

or simply $A_f = -\frac{R_f}{R_g}\ \frac{1}{B}$

The open loop gain of the amplifier is given by

$$A = -\frac{h_{fe}\ R_L}{h_{ie}}$$

Referring to table 2, for OIL 52 transistor, parameters, for h_{fe} and h_{ie} and substituting these values in expression for A

it is found $A \doteq \frac{50 \times 2.7}{1.25} \doteq 108$

Rise time is found to be $= 0.137$ uS which is much less than 0.4 uS the desirable value.

A_f can be adjusted to suit the attenuation of the outputs at various taps, either by varying R_f, R_g or B. But as R_f, also acts as a d.c. path, to the base of the transistor T_1 an alteration in this, means alteration in d.c. bias which is not desirable. So that this has to be done only by altering R_g. Potentiometers are provided for each of the taps and they carry the signal from each of the taps to the respective amplifiers. Hence, by adjusting the potentiometer knobs it is possible to control the output of the amplifier.

Input and output impedances of this amplifier may be seen to be perfectly in order this. Though, the type of feed back that is provided tends to bring down the value of the input impedance, the 10 k base series resistance that is provided is high enough not to present any problem. Output impedance is not affected by a feed back of this type and in fact its value,

$$= 2{,}700\ (1 - \quad) = \frac{2700}{50} = 54 \text{ ohms.}$$

which is a suitable value.

7.3.5. Experimental Results:

1) The amplifier was set up and the d.c. potentials at several points were checked and found to be within expected range of values. Then, signal from each tap was separately fed to the amplifier and potentiometers were adjusted such that the output of the amplifier was the same for all the taps.

2) As inversion is needed for some of the taps, two such amplifiers were used and found to give the necessary inversion. The loading effect of one stage on the other was observed to be negligible.

3) Lastly, tests summation was carried out for some of the taps, by passing the combined amplifier outouts of those taps through another amplifier and result was a satisfactory addition of the pulses. Only some minor adjustments had to be made with the potentiometers to get a flat top while adding 2 or 3 pulses.

7.4. Constructional Details of the Filter Generator Pair:

7.4.1. Circuit diagram of the final assembly:

The circuit diagram given in Figure 7.4. (a) completely describes the filter generator set up.

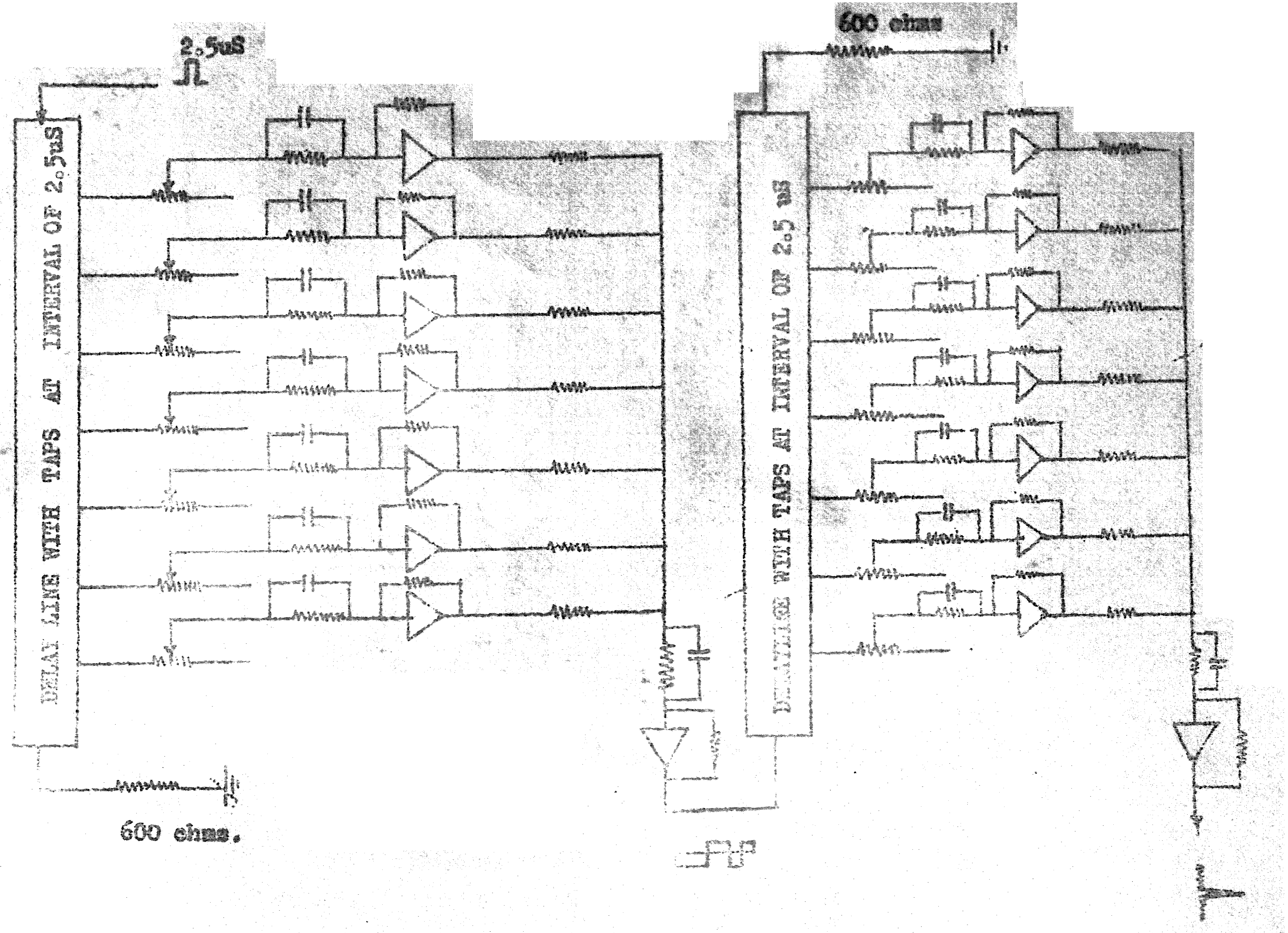
7.4.2. Fabrication:

All the amplifiers needed for the various taps are fabricated on copper clad sheets, properly cut to size, preparing the printed circuitry that is necessary by means of photo resistent etching process. In all, 22 amplifiers are necessary and care is taken to see not to crowd too many transistors on the same sheet as it would affect efficient heat dissipation, the board being a good thermal insulator. Hence, 4 amplifiers are mounted on each piece of copper clad and is referred to as 'gain module'. The necessary summing amplifiers, 2 in number are mounted on a single sheet and is referred to as 'adder module'. ~~These are displayed in plates number I and II.~~

Next, these modules, with their connectors are mounted on a Varipak scheme and are connected to the various taps properly. Now the whole set up represents a compact MF signal and filter pair.

7.4.3. Accessories Used:

1) A pulse generator
2) Cathode Ray oscilloscope type 555
3) Two batteries 6 volts each, for d.c. supply
4) White noise generator
5) Voltmeter



2.5uS
DELAY LINE WITH TAPS AT INTERVAL OF 2.5uS
600 ohms.
600 ohms
WITH TAPS AT INTERVAL OF 2.5 uS

Fig. 7.4 (e)

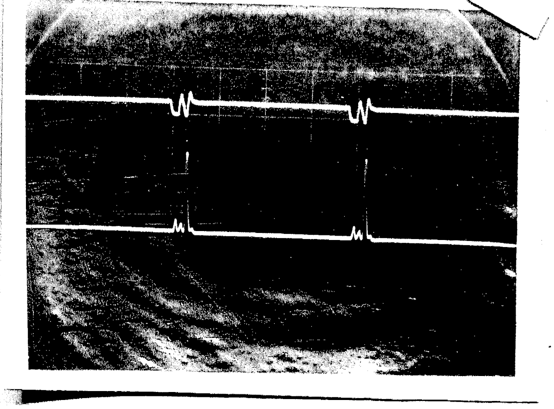


PLATE I

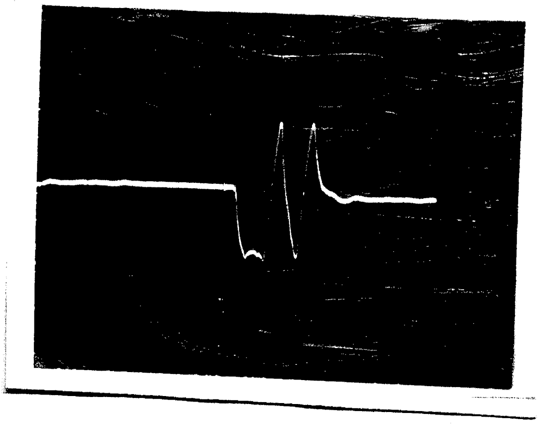


PLATE II

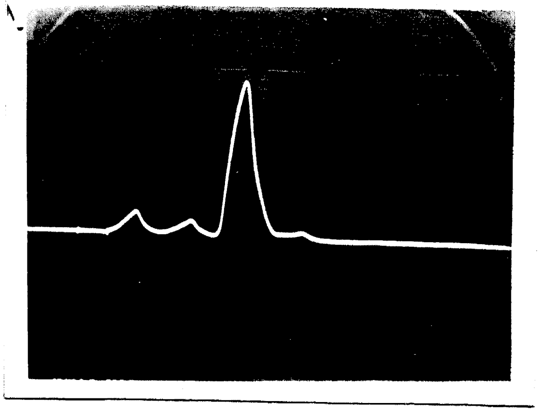


PLATE III

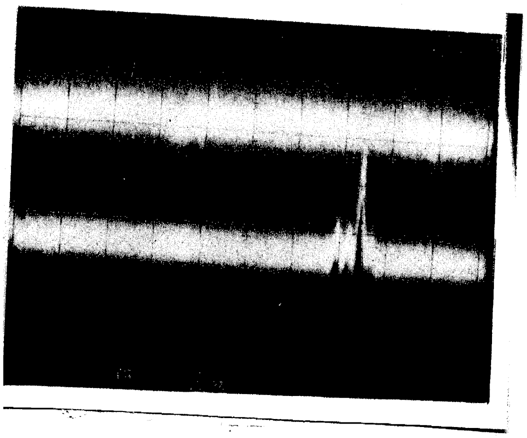


PLATE IV

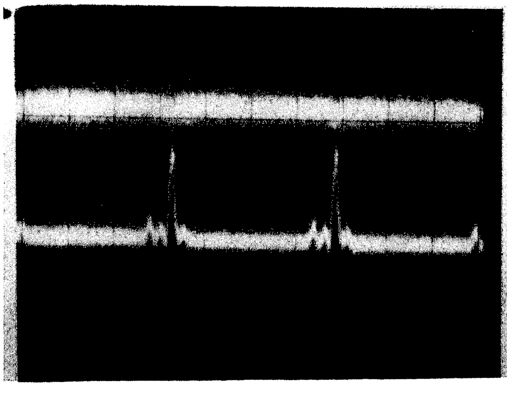


PLATE V

7.4.4. Experimental Results and Observations:

Plates III to VII completely illustrate the performance of the matched signal generator filter set up. These are the photographs of the outputs of generator and filter, displayed on Cathode Ray Oscilloscope type 555, with and without noise.

In plate III the matched signal and its corresponding filter output are shown, employing two beams of the oscilloscope for the purpose. In Plate IV, the matched signal is shown enlarged. In this, ripple at the top that is visible, is caused by the ringing or overshoot of the individual pulses, which is obviously related to the character of the delay lines themselves. Further, while the combination of all the 7 pulses appears to be made up of the sum of pulses with the same absolute value of amplitude, the individual pulses, all of them, do not have the same amplitude. In fact, the overshoots of the combination of pulses, are compensated for by adjusting the gain coefficients or potentiometer knobs slightly.

Plate V shows, the matched filter output. It may be noted that the ratio of peak to clutter, is approximately equal to 6.5 and compares well with the theoretical value which is 7, for a 7 code pseudorandom signal.

However, what is most important is the effect of white noise on the output of MF. This is clearly illustrated in Plate VI. It is important to observe, that while matched signal is completely immersed in noise and is not at all detectable, the MF output, that is the peak, is clearly visible and this is the basic objective of matched filtering which is to filter out noise from the signal even under worst conditions,

and facilitate detection of the signal. The improvement in SNR may be seen to be obvious, in this case.

Plate VII shows the effect of pulse repetition frequency on MF output.

—

8. Conclusions:

This work has covered appreciable amount of several aspects of matched filter theory, generation of pseudorandom codes and construction of matched signal generator and filter pair. This work can be extended in several ways for example a detailed study regarding the various aspects that are to be taken care of while designing filters for signals undergoing Doppler Shifts as in 'tracking' radars, may be made. Further, truncation of long length codes, in a situation where MF and generator can not have the same length of code due to various reasons and its effect on the MF output, require a more detailed study. Effect of "impulse" or "burst" noise on MF output also requires very careful consideration for space applications.

Regarding constructional aspects, many improvements may be made over the set up, realised in this work. For instance, the delay lines that are used, are quite heavy and cumbersome and may be unsuitable in such cases where stress is on reducing the weight of equipment. Also, the amplifiers that are used, may be replaced by integrated circuit modules, which are not only compact but also assure efficiency of operation.

```
$IBFTC CODE
100       FORMAT(I2)
999      READ100,N
         PRINT 250, N
         IF(N.EQ.98)STOP
      DIMENSION A(100),R(100),B(100)
      INTEGER A,R,B
      N1=1
         M=N-1
 40   CALL CGEN(A,N1,N)
         DO 20 I=1,M
         K=N-I
         R(I)=0
         DO 20 J=1,K
         L=J+I
20       R(I)=R(I)+A(J)*A(L)
         RKBAR=0
         DO 25 K=1,M
         XR=R(K)
 25   RKBAR=RKBAR+XR
         XM=M
         RKBAR=RKBAR/XM
         XMU=0
         DO 30 K=1,M
         XR=R(K)
30       XMU=XMU+(XR-RKBAR)**2
         XMU=XMU/XM
         IF(N1.EQ.1)GO TO 35
      IF(XMIN .LT. XMU) GO TO 39
         XMIN=XMU
      PRINT 150, XMU
  50  DO 55 K=1,N
 55   B(K)=A(K)
 39   IF(N1 .NE. (-1)) GO TO 40
      GO TO 41
35       XMIN=XMU
      PRINT 150, XMU
150   FORMAT(1H ,60X,F10.6)
      N1=2
      GO TO 50
 200  FORMAT(1H1,/1X,28HOPTIMUM CODE IS GIVEN BELOW. //)
 250     FORMAT(1H ,20X,14HCODE LENGTH = ,I3//)
      DIMENSION PHI(100)
      DIMENSION S(100)
 41   XS=0
      DO 65 K=1,M
      L=N-K
      JX=0
      DO 60 I=1,K
      IL=I+L
      X=B(I)+B(IL)
      IF(IX.EQ.0)IX=1
      IF(IABS(IX).EQ.2)IX=-1
  60  JX=JX+IX
      X=K
      PHI(K)=JX
```

```
      PHI(K)=(PHI(K)/X-0.5)**2
      XS=XS+PHI(K)
 65   S(K)=XS/X
        DATA NPLUS,MINUS/1H+,1H-/
        DO 45 I=1,N
      IF(B(I).EQ.1) A(I)=NPLUS
      IF(B(I) .EQ. (-1)) A(I)=MINUS
 45     CONTINUE
 300    FORMAT(1H ,30X,100A1)
      PRINT 300,(A(L),L=1,N)
  350 FORMAT(1H ,19HCORRESPONDING MU = ,F8.4,1X)
      PRINT 350,XMIN
 500  FORMAT(1H ,//5X,20F10.5)
C
      GO TO 999
      END
$IBFTC GENR
      SUBROUTINE  CGEN(A,N1,N)
      DIMENSION A(100)
      INTEGER A
      IF(N1 .NE. 1) GO TO 9
      IGEN=2**(N/2)-1
      ILT=IGEN*(2**(N/2+1))
  15  ISTO=IGEN+1
      DO 20 J=1,N
      IREM=IGEN/2
      IREM=IGEN-IREM*2
      IF(IREM.EQ.1) A(J)=+1
      IF (IREM.EQ.0) A(J)=-1
  20  IGEN=IGEN/2
      RETURN
 9    J=ISTO
 10   JJ=J
      JM=J
      IC=0
      DO 30 K=1,N
      IC=IC+JM-JM/2*2
      JM=JM/2
  30  CONTINUE
      IF(IC .EQ. N/2 ) GO TO 35
      J=J+1
      IF(J .LE. ILT) GO TO 10
 35   IGEN=JJ
      IF(IGEN .EQ. ILT) N1=-1
      GO TO 15
      END
```

APPENDIX B

The resistance and capacitence connections to the base of the transistor T_1 of the amplifier, has to be done externally, as these are not included in the gain module printed circuitry. The value of resistance R_1 = 10 k, and the value of the capacitence varies from 18 - 39 pf, from tap to tap.

The procedure to get the matched filter output is, to adjust the gain functions (potentiometers) of generator and filter separately such that the output of the signal generating filter has the desired 7 code form of the matched signal, and that of the matched filter has its time reverse form. Now the two are connected to get the MF output.

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